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PROBLEMS AND SOLUTIONS.

Edited by B. F. FINKEL and OTTO DUNKEL.

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PROBLEMS FOR SOLUTION.

[Errata: *To the number of each problem for solution in the issue of February, 1919, 4 should be added*].

2757. Proposed by ERNEST P. LANE, Rice Institute, Houston, Texas.

Integrate by quadrature the differential equation

$$\frac{d^2y}{dx^2} - 3y\frac{dy}{dx} + y^3 = 0.$$

2758. Proposed by L. RICHARDSON, Vancouver, B. C., Canada.

Prove that, if r be a positive integer,

$$\int_0^{\pi/2} \frac{\sin(2r+1)\psi}{\sin\psi}\,d\psi = \frac{\pi}{2}$$

and

$$\int_0^{\pi/2} \frac{\sin 2r\psi}{\sin\psi}\,d\psi = 2\left\{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^{r-1}}{2r-1}\right\}.$$

2759. Proposed by J. L. RILEY, Stephenville, Texas.

Solve the simultaneous functional equations

$$\Phi(x+y) = \Phi(x) + \frac{\Phi(y)\cdot\psi(x)}{1 - \Phi(x)\Phi(y)},$$

$$\psi(x+y) = \frac{\psi(x)\psi(y)}{1 - \Phi(x)\Phi(y)}.$$

2760. Proposed by CHARLES N. SCHMALL, New York City.

In an arithmetical progression, if s_n be the sum of the first n terms, s_{2n} the sum of the first $2n$ terms, and s_{3n} the sum of the first $3n$ terms of the same series, prove that $s_{2n} - s_n = \frac{1}{3}s_{3n}$.

2761. Proposed by W. W. DENTON, Ann Arbor, Michigan.

Find the length of the side of an equilateral triangle whose vertices are at given distances a, b, c, from a given point.

SOLUTIONS OF PROBLEMS.

411 (Calculus) [June, 1916]. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

Prove that the volume bounded by the surface $f(x, y, z) = 0$ is $1/3\iint (z - x(\partial z/\partial x) - y(\partial z/\partial y))\,dxdy$ integrated over the area determined by projecting the surface on the xy-plane.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

The above problem is not stated accurately for all cases. The integral should be taken over the entire surface,—that is we have a surface integral, where the element of surface, dS, is replaced by its projection on the xy-plane in each element of the sum. This is not the same as integrating over the area described above.

Let us assume that the above surface is a closed, finite one with no double points or singularities. Then the direction cosines of the normal at any point are proportional to $-\partial z/\partial x$, $-\partial z/\partial y$, 1. If we denote the direction cosines by α, β, γ, and remember that $\gamma\cdot dS\cdot = dxdy$, the given integral becomes $1/3\iint(\gamma z + \alpha x + \beta y)dS$ taken over the surface. By a special case of Green's Theorem (B. O. Peirce, *Short Table of Integrals*, formula 883) this becomes $1/3\iiint(1+1+1)dxdydz$, taken throughout the volume inclosed by the surface, which is clearly the volume required.